Centre de recherche sur les transports (C.R.T.)
Centre for Research on Transportation

CRT-2003-01

POINT OF PRESENCE DESIGN IN IP NETWORKS WITH PERFORMANCE GUARANTEES

by

Steven Chamberland

January 2003

Université de Montréal
C.P. 6128, succursale Centre-ville, Montréal QC H3C 3J7 Canada
Téléphone : (514) 343-7575 / Télécopieur : (514) 343-7121

crt@crt.umontreal.ca
www.crt.umontreal.ca

HEC ÉCOLE POLYTECHNIQUE MONTRÉAL Université de Montréal

103-07093/

PUBLICATION
CRT-2003-01

POINT OF PRESENCE DESIGN IN IP NETWORKS WITH PERFORMANCE GUARANTEES

by

Steven Chamberland[1]

This research was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC).

1. Computer Engineering Department, École Polytechnique de Montréal (steven.Chamberland@polymtl.ca), and Centre de recherche sur les transports, Université de Montréal, C.P. 6128, succursale Centre-ville, Montréal, Canada H3C 3J7

Dépôt légal – Bibliothèque nationale du Québec
Bibliothèque nationale du Canada, 2003

Abstract

In this paper, we propose a model for the point of presence (POP) design problem in Internet protocol (IP) networks, where a POP is a node composed of several interconnected co-located backbone routers. This problem consists of selecting the number of routers and their types, selecting the interface card types, connecting the access and the backbone links to the ports and selecting the link types between the co-located routers. The model also consider the routing of the IP traffic. The performance guarantees we refer to are bandwidth guarantees between the routers for the normal state of the POP and also for all failure scenarios of interest to the network planner. Since this problem is $\mathcal{NP}$-hard, we propose a tabu search heuristic to find solutions for real-size instances of the problem. Finally, we present a systematic set of experiments designed to assess the performance of the proposed heuristic. The results show that quasi-optimal solutions can be found with the proposed heuristic.

Keywords: Internet protocol (IP) networks; point of presence (POP) design; mathematical model; tabu search.

Résumé

Dans cet article, nous traitons du problème de la conception des points de présence dans les réseaux IP (Internet Protocol). Ce problème consiste à sélectionner le nombre de routeurs et le type de chacun, sélectionner les types de cartes d'interface à insérer dans les routeurs, brancher les liens d'accès et de transmission aux ports et sélectionner les types de liens entre les routeurs co-localisés. Un modèle de programmation mathématique à variables mixtes est proposé pour ce problème ainsi qu'une heuristique basée sur le principe de la recherche avec tabous pour trouver de bonnes solutions.

Mots-clés: Réseaux IP; point de présence; conception; modèle mathématique; recherche avec tabous.

1 Introduction

Due to the increasing demand for Internet network services, telecommunication service providers are still investing in their Internet protocol (IP) infrastructure. As a result, several backbone routers are co-located in the same central office forming large IP point of presence (POP). Considering that IP routers and interface cards are still expensive, POP optimization is an important issue for these providers.

Actually, the POP planning is typically done by hand or by using a network simulator to evaluate several scenarios. Since the number of routers increased continuously in a POP, the "what if" planning approaches are not optimal anymore. In this paper, we introduce the POP design problem in IP networks with performance guarantees that consists of

- selecting the number of routers and their types (where a type of router is characterized by its number of slots and its switch fabric capacity) to install in the POP;
- selecting the interface card types (where an interface card type is characterized by its technology, the port rate and the number of ports per card);
- connecting the access and backbone links to the ports;
- connecting the network of the co-located routers.

The objective is to minimize the cost of the POP. The model also consider the routing of the IP traffic using a standardized routing protocol (e.g., the open shortest path first (OSPF) routing protocol). The performance guarantees we refer to are bandwidth guarantees between the routers for the normal state of the POP and also for all failure scenarios of interest to the network planner.

More precisely, we make the following assumptions about the organization of each POP: (A1) The POP is composed of co-located interconnected backbone routers; (A2) The number of routers installed in the POP cannot exceed the maximum allowed; (A3) The number of interface cards installed in a router cannot exceed its number of slots; (A4) The sum of the rate of the ports installed in a router cannot exceed its switch fabric capacity. We also assume that the following information is known: (F1) The access link types (where an access link type is characterized by its technology and rate) to connect to the POP; (F2) The backbone links types (where a backbone link type is characterized by its technology and rate) to connect to the POP; (F3) The maximum number of routers that can be installed in the POP; (F4) The different types

of routers and interface cards; (F5) The cost of each router type and the cost of installing it (including the floor space, cables, racks, electric installation and labor); (F6) The cost of each interface card type and the cost of installing it (including the patch panel, the cables and labor); (F7) The cost of a link of a given type used to interconnect two co-located routers and the cost of installing it (including the labor); (F8) The cost of connecting an access and a backbone link of a given type to a router in the POP (including the cables and labor); (F9) The bandwidth requirement between the service points; (F10) The network failure scenarios of interest to the network planner. The problem consists to find the minimum cost POP subject to all of the above assumptions (A1 to A4) and facts (F1 to F10).

The literature of network engineering and operations research contains many articles relating to network optimization and planning, but the POP design problem defined here was not considered. The most related problems concern the two-level network design problem (see [1, 2, 3] and the references contained therein). However, these models did not consider simultaneously the following features necessary in the POP design problem:

- the two-level topological network design problem;
- the network dimensioning problem;
- the use of modular switches with different access and backbone transport technologies and several ports per interface card.

This paper is organized as follows. Section 2 presents a mathematical model for the POP design problem in IP networks. Section 3 presents a tabu based heuristic design to find a good solution of the model and Section 4 presents a systematic set of experiments designed to assess the performance of the proposed heuristic (using a lower bound). Conclusions and further works are presented in Section 5.

2 The Modeling Framework

2.1 The Notation

The following notation is used throughout this paper. The notation is composed of sets, decision variables, traffic variables, cost parameters and constants.

Sets: Let M be the set of clients and N, the set of other POPs (denoted O-POP). Let R be the set of link and port types (where α_r is the bit rate (in bps) of the link and port of type $r \in R$) such that $R = R^A \cup R^B \cup R^P$ where R^A is the set of link and port types used in the access network, R^B the set of link and port types used in the backbone network and R^P, the set of link and port types used to interconnect the routers in the POP. Let I be the set of access links such that $I = \bigcup_{r \in R^A} I_r$ where I_r is the set of access links of type $r \in R^A$ and $I = \bigcup_{m \in M} I_m$ where I_m is the set of access links from client $m \in M$ and J, the set of backbone links such that $J = \bigcup_{r \in R^B} J_r$ where J_r is the set of backbone links of type $r \in R^B$ and $J = \bigcup_{n \in N} J_n$ where J_n is the set of backbone links from O-POP $n \in N$. Let K be the set of potential routers' locations in the POP and S, the set of network states that include the no failure scenario (i.e., the state s_0) and the failure scenarios that the network planner wants to take into account (i.e., the states s_1 to $s_{|S|-1}$). Finally, let T, the set of router types (where m^t is the number of slots in a router of type $t \in T$ and β^t its switch fabric capacity) and L, the set of interface card types (where n^l is the number of ports on the card of type $l \in L$) such that $L = \cup_{r \in R} L_r$ where L_r is the set of interface cards with ports of type $r \in R$.

Decision Variables: Let u_k^t be a 0-1 variable such that $u_k^t = 1$ if and only if the router installed at location $k \in K$ is of type $t \in T$ and v_k^l, the number of cards of type $l \in L$ installed at location $k \in K$. Let x_{ik} be a 0-1 variable such that $x_{ik} = 1$ if and only if the access link $i \in I$ is connected to location $k \in K$, y_{jk}, a 0-1 variable such that $y_{jk} = 1$ if and only if the backbone link $j \in J$ is connected to location $k \in K$ and $z_{kk'}^r$, the number of links of type $r \in R^P$ installed between location $k \in K$ and location $k' \in K$ (for $k < k'$).

Traffic Variables: Let f_{mk}^{os} (f_{km}^{os}) be the traffic flow (in bps) from client $m \in M$ to location $k \in K$ (from location $k \in K$ to client $m \in M$) originating from $o \in M \cup N$ in the network state $s \in S$; f_{nk}^{os} (f_{kn}^{os}), the traffic flow (in bps) from O-POP $n \in N$ to location $k \in K$ (from location $k \in K$ to O-POP $n \in N$) originating from $o \in M \cup N$ in the network state $s \in S$ and finally, $f_{kk'}^{os}$, the traffic flow (in bps) from location $k \in K$ to location $k' \in K$ originating from $o \in M \cup N$ in the network state $s \in S$.

Cost Parameters: Let a^l be the cost of a card of type $l \in L$ (including the installation cost) and b^t be the cost of a router of type $t \in T$ (including the installation cost), c^r, the cost of a link of type $r \in R^P$ (including the installation cost) to interconnect two co-located routers. Finally, let d^r be the cost of connecting an access and backbone link of type $r \in R^A \cup R^B$ to a router in the POP (including the installation cost).

Constants: Let p^{od} be the bandwidth requirement from $o \in M \cup N$ to $d \in (M \cup N) \setminus \{o\}$ passing through the POP (in bps) and q_m^r (q_n^r), the number of links of type $r \in R$ from $m \in M$ (from $n \in N$) to the POP.

2.2 The Cost Function

The objective function, representing the total cost of the POP, is composed of four terms: the cost of the interface cards, the cost of the routers, the cost of the inter-router links and the cost of the access and backbone links. The cost of the interface cards, noted C_I, includes the cost of the cards and installing them.

$$C_I(\mathbf{v}) = \sum_{l \in L} a^l \sum_{k \in K} v_k^l. \tag{1}$$

The cost of the routers, noted C_R includes the cost of the routers and installing them.

$$C_R(\mathbf{u}) = \sum_{t \in T} b^t \sum_{k \in K} u_k^t \tag{2}$$

The cost of the inter-router links, noted C_{IL} given by the following equation, includes the cost of the links and installing them.

$$C_{IL}(\mathbf{z}) = \sum_{r \in R^p} c^r \sum_{k \in K} \sum_{\substack{k' \in K \\ k < k'}} z_{kk'}^r. \tag{3}$$

The cost of the access and backbone links, noted $C_{A\&BL}$ given by the following equation.

$$C_{A\&BL}(\mathbf{x}, \mathbf{y}) = \sum_{r \in R^A} d^r \sum_{i \in I_r} \sum_{k \in K} x_{ik} + \sum_{r \in R^B} d^r \sum_{j \in J_r} \sum_{k \in K} y_{jk}. \tag{4}$$

2.3 The Mathematical Model

In this section, we define the POP design problem in IP networks with performance guarantees, denote P. This model consists of minimizing the cost function subject to constraints. Observe that the inter-router network topology has not yet specified as well as the routing mechanism and the failure scenarios considered.

P:

$$\min_{\mathbf{f},\mathbf{u},\mathbf{v},\mathbf{x},\mathbf{y},\mathbf{z}} C_I(\mathbf{v}) + C_R(\mathbf{u}) + C_{IL}(\mathbf{z}) + C_{A\&BL}(\mathbf{x}, \mathbf{y}) \tag{5}$$

subject to

Access link assignment constraints

$$\sum_{k \in K} x_{ik} = 1 \quad (i \in I_r, r \in R^A) \tag{6}$$

Backbone link assignment constraints

$$\sum_{k \in K} y_{jk} = 1 \quad (j \in J_r, r \in R^B) \tag{7}$$

Router-type uniqueness constraints

$$\sum_{t \in T} u_k^t \leq 1 \quad (k \in K) \tag{8}$$

Router-capacity constraints (slot level)

$$\sum_{l \in L} v_k^l \leq \sum_{t \in T} m^t u_k^t \quad (k \in K) \tag{9}$$

Router-capacity constraints (switch fabric level)

$$\sum_{r \in R} \alpha_r \sum_{l \in L_r} n^l v_k^l \leq \sum_{t \in T} \beta^t u_k^t \quad (k \in K) \tag{10}$$

Router-capacity constraints (port level)

$$\sum_{i \in I} x_{ik} \leq \sum_{l \in L_r} n^l v_k^l \quad \left(i \in I_r, r \in R^A \setminus \left(R^B \cup R^P\right), k \in K\right) \tag{11}$$

$$\sum_{j \in J} y_{jk} \leq \sum_{l \in L_r} n^l v_k^l \quad \left(j \in J_r, r \in R^B \setminus \left(R^A \cup R^P\right), k \in K\right) \tag{12}$$

$$\sum_{\substack{k' \in K \\ k < k'}} z_{kk'}^r + \sum_{\substack{k' \in K \\ k > k'}} z_{k'k}^r \leq \sum_{l \in L_r} n^l v_k^l \quad \left(r \in R^P \setminus \left(R^A \cup R^B\right), k \in K\right) \tag{13}$$

$$\sum_{i \in I} x_{ik} + \sum_{j \in J} y_{jk} \leq \sum_{l \in L_r} n^l v_k^l \quad \left(i \in I_r, j \in J_r, r \in \left(R^A \cap R^B\right) \setminus R^P, k \in K\right) \tag{14}$$

$$\sum_{i \in I} x_{ik} + \sum_{\substack{k' \in K \\ k < k'}} z_{kk'}^r + \sum_{\substack{k' \in K \\ k > k'}} z_{k'k}^r \leq \sum_{l \in L_r} n^l v_k^l \quad \left(i \in I_r, r \in \left(R^A \cap R^P\right) \setminus R^B, k \in K\right) \tag{15}$$

$$\sum_{j \in J} y_{jk} + \sum_{\substack{k' \in K \\ k < k'}} z_{kk'}^r + \sum_{\substack{k' \in K \\ k > k'}} z_{k'k}^r \leq \sum_{l \in L_r} n^l v_k^l \quad \left(j \in J_r, r \in \left(R^B \cap R^P\right) \setminus R^A, k \in K\right) \tag{16}$$

$$\sum_{i \in I} x_{ik} + \sum_{j \in J} y_{jk} + \sum_{\substack{k' \in K \\ k < k'}} z_{kk'}^r + \sum_{\substack{k' \in K \\ k > k'}} z_{k'k}^r \leq \sum_{l \in L_r} n^l v_k^l \quad \left(i \in I_r, j \in J_r, r \in R^A \cap R^B \cap R^P, k \in K\right) \tag{17}$$

Inter-router network topology constraints

$$\boxed{\text{Inter-router network topology constraints}} \tag{18}$$

Access link capacity constraints

$$f_{mk}^{ms} \leq \sum_{r \in R^A} \sum_{i \in I_m \cap I_r} \alpha_r x_{ik} \quad (m \in M, k \in K, s \in S) \tag{19}$$

$$\sum_{o \in (M \setminus \{m\}) \cup N} f_{km}^{os} \leq \sum_{r \in R^A} \sum_{i \in I_m \cap I_r} \alpha_r x_{ik} \quad (m \in M, k \in K, s \in S) \tag{20}$$

Backbone link capacity constraints

$$f_{nk}^{ns} \leq \sum_{r \in R^B} \sum_{j \in J_n \cap J_r} \alpha_r y_{jk} \quad (n \in N, k \in K, s \in S) \tag{21}$$

$$\sum_{o \in M \cup (N \setminus \{N\})} f_{kn}^{os} \leq \sum_{r \in R^B} \sum_{j \in J_n \cap J_r} \alpha_r y_{jk} \quad (n \in N, k \in K, s \in S) \tag{22}$$

Inter-router link capacity constraints

$$\sum_{o \in M \cup N} f_{kk'}^{os} \leq \sum_{r \in R^P} \alpha_r z_{kk'}^r \quad (k < k', k, k' \in K, s \in S) \tag{23}$$

$$\sum_{o \in M \cup N} f_{k'k}^{os} \leq \sum_{r \in R^P} \alpha_r z_{kk'}^r \quad (k < k', k, k' \in K, s \in S) \tag{24}$$

Traffic flow conservation constraints

$$\sum_{k \in K} f_{mk}^{ms} = \sum_{d \in (M \setminus \{m\}) \cup N} p^{md} \quad (m \in M, s \in S) \tag{25}$$

$$\sum_{k \in K} f_{km}^{os} = p^{om} \quad (o \in (M \setminus \{m\}) \cup N, m \in M, s \in S) \tag{26}$$

$$\sum_{k \in K} f_{nk}^{ns} = \sum_{d \in M \cup (N \setminus \{n\})} p^{nd} \quad (n \in N, s \in S) \tag{27}$$

$$\sum_{k \in K} f_{kn}^{os} = p^{on} \quad (o \in M \cup (N \setminus \{n\}), n \in N, s \in S) \tag{28}$$

$$\sum_{m \in M} (f_{mk}^{os} - f_{km}^{os}) + \sum_{n \in N} (f_{nk}^{os} - f_{kn}^{os}) + \sum_{k' \in K \setminus \{k\}} (f_{k'k}^{os} - f_{kk'}^{os}) = 0 \quad (o \in M \cup N, k \in K, s \in S) \tag{29}$$

Routing constraints

Routing constraints (30)

Failure scenarios constraints

Failure scenarios constraints (31)

Integrality and nonnegativity constraints

$$\mathbf{f} \in \mathbf{R}_{+}^{|S||K|(|M|+|N|)(|M|+|N|+|K|-1)}, \mathbf{u} \in \{0,1\}^{|K||T|}, \mathbf{v} \in \mathbf{N}^{|K||L|},$$
$$\mathbf{x} \in \{0,1\}^{|I||K|}, \mathbf{y} \in \{0,1\}^{|J||K|}, \mathbf{z} \in \mathbf{N}^{\frac{|K|}{2}(|K|-1)|R^P|}. \tag{32}$$

The objective function (5) of P, as mentioned before, is composed of four terms, representing the total cost of the POP. Constraints (6) are access link assignment constraints that require each access link be connected to the POP and constraints (7) are the backbone link assignment constraints that require each backbone link be connected to the POP. The router-type uniqueness constraints (8) impose that at most one router type be installed at location $k \in K$ and constraints (9) necessitate the total number of slots used by the interface cards at location $k \in K$ be less than or equal to the number of slots available in the router type installed at this location. Constraints (10) impose the sum of the rates of the ports installed in the router at location $k \in K$ be less than or equal to its switch fabric capacity and constraints (11) to (17) dictate the number of links of type $r \in R$ connected to a router be less than or equal to the number of ports of this type installed in this router.

Constraints (18) necessitate that the topology of the inter-router network be the one specified by the network planner. The full-mesh topology is considered in this paper. This topology can be described using the following set of constraints.

$$\sum_{t \in T} u_k^t + \sum_{t \in T} u_{k'}^t \leq \sum_{r \in R^P} z_{kk'}^r + 1 \quad (k < k', k, k' \in K). \tag{33}$$

These constraints were described by Chung, Myung and Tcha [4] and require at least one link between two locations if and only if routers have been installed at these two locations. Other topologies would have been considered such as the tree, ring [1] or multiple ring topology [2].

Access link capacity constraints (19) and (20) impose the traffic flow (in bps) from client $m \in M$ to the POP be less than or equal the total access link capacities from this client (in both directions) and backbone link capacity constraints (21) and (22) necessitate the traffic flow from O-POP $n \in N$ to the POP be less or equal to the total backbone link capacities installed from this O-POP (in both directions). Inter-router link capacity constraints (23) and (24) impose the traffic flow (in bps) from location $k \in J$ to location $k' \in K$ be less or equal to the total link capacities installed between k and k'. Constraints (25) to (29) are traffic

flow conservation constraints and constraints (30) are the routing constraints. Since a POP is an autonomous system, the open shortest path first (OSPF) routing mechanism is used in this paper to route the traffic in the POP (see Moy [9] for more details about OSPF).

Constraints (31) are the failure scenarios constraints such as the single inter-router link or router failure scenarios constraints. Consider that the network planner wants to consider the single link failure scenarios in P. If we note

$$\{(k,k') : k \neq k', k, k' \in K\} = \left\{(k_1,k'_1),(k_2,k'_2),\ldots,(k_{|K|(|K|-1)},k'_{|K|(|K|-1)})\right\},$$

and suppose that link (k_l, k'_l) is failed in the network state s_l for $l = 1$ to $|K|(|K|-1)$, constraints (31) are

$$f^{os_1}_{k_1k'_1} = 0, f^{os_2}_{k_2k'_2} = 0, ..., f^{os_{|K|(|K|-1)}}_{k_{|K|(|K|-1)}k'_{|K|(|K|-1)}} = 0 \quad (o \in I \cup J) \tag{34}$$

These constraints impose, for each failure scenario, the traffic to pass around the failed link (in one direction). Constraints (32) are integrality and nonnegativity constraints.

Proposition. P is $\mathcal{NP}$-hard.

Proof. Transformation from the knapsack problem (for more details about the knapsack problem, see Nemhauser and Wolsey [7]).

Since P is $\mathcal{NP}$-hard, in the rest of this paper, we concentrate our efforts on the development of efficient heuristics.

3 The Tabu Search Algorithm

In this section, we propose a tabu search (TS) algorithm for P, called TS. An introduction to tabu search can be found in [5].

The following notation and definitions will be used to describe TS. Let e_k be the state of location $k \in K$ such that $e_k = 0$ if there is no router installed at location k, and $e_k = t$ (for $t \in T = \{1, ..., |T|\}$), if a router of type t is installed at location k. Let $\mathbf{e} = \{e_k\}_{k \in K}$ be the vector of the state of the POP location. Therefore, when the vector $\mathbf{e}$ is fixed, the variables $\mathbf{u}$ are also fixed. Let $\mathbf{u}(\mathbf{e})$ be the vector $\mathbf{u}$ when the POP location state $\mathbf{e}$ is fixed.

In the next subsection we propose a decomposition approach to solve P($\mathbf{e}$), i.e., the model P when the decision vector $\mathbf{u}$ equals $\mathbf{u}(\mathbf{e})$.

3.1 Solving P(e)

When vector $\mathbf{u}$ is set to $\mathbf{u}(\mathbf{e})$, P can be decomposed into two subproblems. First, consider that h_k slots are temporary reserved in location k to install interface cards for the inter-router links. Therefore, the following constraints apply.

$$\sum_{l \in L} v_k^l \leq \sum_{t \in T} \max\left\{\left(m^t - h_k\right), 0\right\} u_k^t \quad (k \in K) \tag{35}$$

and

$$\sum_{r \in R} \alpha_r \sum_{l \in L_r} n^l v_k^l \leq \sum_{t \in T} \max\left\{\left(\beta^t - h_k \alpha_{r^*} \max_{l \in L_{r^*}}\left\{n^l\right\}\right), 0\right\} u_k^t \quad (k \in K) \tag{36}$$

where $r^* := \arg\max_{r \in R^P}\{\alpha_r\}$.

3.1.1 The First Subproblem

The first subproblem, noted $\overline{\mathrm{P}}(\mathbf{u}, \mathbf{h})$, is given below.

$\overline{\mathrm{P}}(\mathbf{u}, \mathbf{h})$:

$$\min_{\mathbf{v},\mathbf{x},\mathbf{y}} C_{A\&BL}(\mathbf{x}, \mathbf{y}) + \sum_{r \in R^A \cup R^B} \sum_{l \in L_r} a^l \sum_{k \in K} v_k^l \tag{37}$$

subject to (6)-(7), (35)-(36) and

$$\sum_{i \in I} x_{ik} \leq \sum_{l \in L_r} n^l v_k^l \quad (i \in I_r, r \in R^A \setminus R^B, k \in K) \tag{38}$$

$$\sum_{j \in J} y_{jk} \leq \sum_{l \in L_r} n^l v_k^l \quad (j \in J_r, r \in R^B \setminus R^A, k \in K) \tag{39}$$

$$\sum_{i \in I} x_{ik} + \sum_{j \in J} y_{jk} \leq \sum_{l \in L_r} n^l v_k^l \quad (i \in I_r, j \in J_r, r \in R^A \cap R^B, k \in K) \tag{40}$$

$$v_k^l \in \mathbf{N} \quad (k \in K, l \in L_r, r \in R^A \cup R^B) \tag{41}$$

$$\mathbf{x} \in \{0,1\}^{|I||K|}, \mathbf{y} \in \{0,1\}^{|J||K|}. \tag{42}$$

The purpose of this subproblem is to connect the access and backbone links to the routers and selecting the interface cards to connect them at minimum cost considering the router types chosen (given by vector $\mathbf{u}$) and the slots reserved for the inter-router links (given by vector $\mathbf{h}$). Since this subproblem is $\mathcal{NP}$-hard (transformation from the knapsack problem) and we may have to solve several thousands instances of it during the search process, we propose a heuristic to find a good solution quickly. This heuristic, called HFS (Heuristic for the First Subproblem) is defined below.

Algorithm HFS

Step 1: (Initialisation)

1.1 Set $v_k^l := 0$ for all $k \in K, l \in L_r, r \in R^A \cup R^B$, $\mathbf{x} := \mathbf{0}$ and $\mathbf{y} := \mathbf{0}$.

1.2 Set $l(r) := \arg\max_{l \in L_r}\{n^l\}$ for all $r \in R^A \cup R^B$.

1.3 Calculate NumSlotsLeft(k) defined as the number of slots left for the access and backbone links at location $k \in K$ and CapacityLeft(k), the capacity left (in bps) at the switch fabric level for those links. Therefore,

$$\text{NumSlotsLeft}(k) := \sum_{t \in T} \max\left\{\left(m^t - h_k\right), 0\right\} u_k^t \quad (k \in K)$$

$$\text{CapacityLeft}(k) := \sum_{t \in T} \max\left\{\left(\beta^t - h_k \alpha_{r^*} \max_{l \in L_{r^*}}\left\{n^l\right\}\right), 0\right\} u_k^t \quad (k \in K).$$

1.4 Define $P = \{p_1, p_2, ..., p_{|P|}\}$ a subset of K defined as the set of routers' locations in the POP such that NumSlotsLeft(p_1) $\geq$ NumSlotsLeft(p_2) $\geq \ldots \geq$ NumSlotsLeft($p_{|P|}$) > 0.

Step 2: (Connect the backbone links to the routers)

2.1 Order the elements in R^B such that $\alpha_{r_1^B} > \alpha_{r_2^B} > \ldots > \alpha_{r_{|R^B|}^B}$ and $r_1^B \prec r_2^B \prec \ldots \prec r_{|R^B|}^B$.

2.2 Calculate MinNumSlotsB(r) defined as the minimum number of slots necessary to connect the backbone links of type $r \in R^B$. Therefore,

$$\text{MinNumSlotsB}(r) := \left\lceil \frac{|J_r|}{n^{l(r)}} \right\rceil \quad (r \in R^B).$$

2.3 Calculate MinNumRoutersB the minimum number of routers necessary to insert the interface cards used for the backbone links. Therefore,

$$\text{MinNumRoutersB} := \arg \min_{n \in \{1,2,\ldots,|P|\}} \left\{ n : \sum_{p=p_1}^{p_n} \text{NumSlotsLeft}(p) \geq \sum_{r \in R^B} \text{MinNumSlotsB}(r) \right\}.$$

2.4 For $n :=$ MinNumRoutersB to $|P|$ do

2.4.1 Set $P(n) := \{p_1, p_2, ..., p_n\}$.

2.4.2 For $r := r_1^B$ to $r_{|R^B|}^B$ do

Solve the following subproblem, called $\overline{\text{PB}}_{r,n}(\mathbf{h}, \mathbf{u}, \mathbf{y})$, that consists to connect the backbone links to the routers in $P(n)$ considering that the backbone links $\bigcup_{r' \in R^B : r' \prec r} J_r$ are already connected to routers.

$\overline{\text{PB}}_{r,n}(\mathbf{h}, \mathbf{u}, \mathbf{y})$:

$$\min_{\{y_{jk} : j \in J_r, k \in P(n)\}} \sum_{j \in J_r} \sum_{k \in P(n)} d^r y_{jk} \tag{43}$$

subject to

$$\sum_{k \in P(n)} y_{jk} = 1 \quad (j \in J_r) \tag{44}$$

$$\sum_{j \in J_r} y_{jk} \leq \min \left\{ n^{l(r)} \left(\text{NumSlotsLeft}(k) - \sum_{r' \in R^B} v_k^{l(r')} \right), \right.$$
$$\left. n^{l(r)} \left\lfloor \frac{1}{\alpha_r n^{l(r)}} \left(\text{CapacityLeft}(k) - \sum_{r' \in R^B} \alpha_{r'} n^{l(r')} v_k^{l(r')} \right) \right\rfloor \right\} \quad (k \in P(n)) \tag{45}$$

$$y_{jk} \in \{0, 1\} \quad (j \in J_r, k \in P(n)). \tag{46}$$

This subproblem is an instance of the linear assignment problem and to solve it, we use the shortest augmenting path algorithm LAPJV of Jonker and Volgenant [6]. If $\overline{\text{PB}}_{r,n}(\mathbf{h}, \mathbf{u}, \mathbf{y})$ is feasible set $v_k^{l(r)} := \left\lceil \frac{\sum_{j \in J_r} y_{jk}}{n^{l(r)}} \right\rceil$ for all $k \in P(n)$.

2.4.3 If $\overline{\text{PB}}_{r,n}(\mathbf{h}, \mathbf{u}, \mathbf{y})$ is feasible for all $r \in R^B$ go to Step 3. Otherwise go to Step 2.4.4.

2.4.4 If $n = |P|$, stop, the subproblem $\overline{\text{P}}(\mathbf{u}, \mathbf{h})$ is not feasible. Otherwise continue.

Step 3: (Connect the access links to the routers)

3.1 Order the elements in R^A such that $\alpha_{r_1^A} > \alpha_{r_2^A} > ... > \alpha_{r_{|R^A|}^A}$ and $r_1^A \prec r_2^A \prec \ldots \prec r_{|R^A|}^A$.

3.2 For $r := r_1^A$ to $r_{|R^A|}^A$ do

Solve the following subproblem, called $\overline{\text{PA}}_r(\mathbf{h}, \mathbf{u}, \mathbf{v}, \mathbf{x}, \mathbf{y})$, that consists to connect the access links to the routers in P considering that the backbone links are already connected to routers as well as the access links $\bigcup_{r' \in R^A : r' \prec r} I_r$.

$\overline{\text{PA}}_r(\mathbf{h}, \mathbf{u}, \mathbf{v}, \mathbf{x}, \mathbf{y})$:

$$\min_{\{x_{ik} : i \in I_r, k \in P\}} \sum_{i \in I_r} \sum_{k \in P} d^r x_{ik} \tag{47}$$

subject to

$$\sum_{k \in P} x_{ik} = 1 \quad (i \in I_r) \tag{48}$$

$$\begin{aligned} \sum_{i \in I_r} x_{ik} \quad \leq \quad & \min \left\{ n^{l(r)} \left(\text{NumSlotsLeft}(k) - \sum_{r' \in R^A \setminus R^B \cup R^B} v_k^{l(r')} \right), \right. \\ & \left. n^{l(r)} \left\lfloor \frac{1}{\alpha_r n^{l(r)}} \left(\text{CapacityLeft}(k) - \sum_{r' \in R^A \setminus R^B \cup R^B} \alpha_{r'} n^{l(r')} v_k^{l(r')} \right) \right\rfloor \right\} \\ & + \sum_{r' \in R^B \cap \{r\}} \left(n^{l(r')} v_k^{l(r')} - \sum_{j \in J_{r'}} y_{jk} \right) \quad (k \in P) \end{aligned} \tag{49}$$

$$x_{ik} \in \{0, 1\} \quad (i \in I_r, k \in P). \tag{50}$$

This subproblem is an instance of the linear assignment problem and to solve it, we use the shortest augmenting path algorithm LAPJV of Jonker and Volgenant [6]. If $\overline{\text{PA}}_r(\mathbf{h}, \mathbf{u}, \mathbf{v}, \mathbf{x}, \mathbf{y})$ is feasible set

$$v_k^{l(r)} := v_k^{l(r)} + \left\lceil \frac{\max \left\{ \sum_{i \in I_r} x_{ik} - \left(\sum_{r' \in R^B \cap \{r\}} \left(n^{l(r')} v_k^{l(r')} - \sum_{j \in J_r} y_{jk} \right) \right), 0 \right\}}{n^{l(r)}} \right\rceil$$

for all $k \in P$.

3.1.2 The Second Subproblem

The second subproblem, noted $\overline{\overline{\text{P}}}(\mathbf{h}, \mathbf{u}, \mathbf{v}, \mathbf{x}, \mathbf{y})$, is given below.

$\overline{\overline{\text{P}}}(\mathbf{h}, \mathbf{u}, \mathbf{v}, \mathbf{x}, \mathbf{y})$:

$$\min_{\mathbf{f}, \mathbf{v}, \mathbf{z}} C_{IL}(\mathbf{z}) + \sum_{r \in R^P} \sum_{l \in L_r} a^l \sum_{k \in K} v_k^l \tag{51}$$

subject to (9)-(10), (13), (15)-(31) and

$$v_k^l \in \mathbf{N} \quad (k \in K, l \in L_r, r \in R^P) \tag{52}$$

$$\mathbf{f} \in \mathbf{R}_+^{|S||K|(|M|+|N|)(|M|+|N|+|K|-1)}, \mathbf{z} \in \mathbf{N}^{\frac{|K|}{2}(|K|-1)|R^P|}. \tag{53}$$

The purpose of the above subproblem is to connect the co-located routers and selecting the interface cards to connect them at minimum cost. The following algorithm, called HSS (Heuristic for the Second Subproblem), is used to solve this subproblem.

Algorithm HSS

Step 1: (Initialisation)

1.1 Set $\mathbf{f} := \mathbf{0}$, $v_k^l := 0$ for all $k \in K, l \in L_r, r \in R^P$ and $\mathbf{z} := \mathbf{0}$.

1.2 Set $l(r) := \arg\max_{l \in L_r}\{n^l\}$ for all $r \in R^P$ and $r^\star := \arg\max_{r \in R^P}\{\alpha_r\}$.

1.3 Define $P = \{p_1, p_2, ..., p_{|P|}\}$ a subset of K defined as the set of routers' locations in the POP.

Step 2: (Find the topology of the inter-router network)

Find the minimum cost inter-router topology respecting constraints (18) and the degree constraints B_k for all $k \in K$, where

$$\begin{aligned} B_k \;=\; \min \Bigg\{ & n^{l(r^\star)} \left(\sum_{t \in T} m^t u_k^t - \sum_{r' \in R^A \setminus R^B \cup R^B} v_k^{l(r')} \right), \\ & \left\lfloor \frac{1}{\alpha_{r^\star} n^{l(r^\star)}} \left(\sum_{t \in T} \beta^t u_k^t - \sum_{r' \in R^A \setminus R^B \cup R^B} \alpha_{r'} n^{l(r')} v_k^{l(r')} \right) \right\rfloor \Bigg\} \\ & + \sum_{r' \in R^A \setminus R^B \cap \{r^\star\}} \left(n^{l(r')} v_k^{l(r')} - \sum_{i \in I_{r'}} x_{ik} \right) \\ & + \sum_{r' \in R^B \setminus R^A \cap \{r^\star\}} \left(n^{l(r')} v_k^{l(r')} - \sum_{j \in J_{r'}} y_{jk} \right) \\ & + \sum_{r' \in R^B \cap R^A \cap \{r^\star\}} \left(n^{l(r')} v_k^{l(r')} - \sum_{i \in I_{r'}} x_{ik} - \sum_{j \in J_{r'}} y_{jk} \right) \quad (k \in P). \end{aligned} \tag{54}$$

(In this paper, the full-mesh topology is considered. As a result, there is an inter-router link between each pair of routers if $B_k \geq (|P| - 1)$ for all $k \in P$. Otherwise the subproblem $\overline{\overline{\mathrm{P}}}(\mathbf{h}, \mathbf{u}, \mathbf{v}, \mathbf{x}, \mathbf{y})$ is not feasible.)

Step 3: (Evaluate the maximum traffic on each inter-router link)

For each scenario $s \in S$, find the traffic flow (in bps) on each link using a routing simulator considering the topology found in Step 2. (In this paper, an OSPF routing simulator is used.)

Step 4: (Inter-router link dimensioning)

For each link $k, k' \in P$ $(k < k')$ find the minimum value of $z^{r^\bullet}_{kk'}$ such that

$$\alpha_{r^\bullet} z^{r^\bullet}_{kk'} \geq \max_{s \in S} \left\{ \sum_{o \in M \cup N} f^{os}_{kk'}, \sum_{o \in M \cup N} f^{os}_{k'k} \right\}. \tag{55}$$

Step 5: For all $k \in P$ do

$$\begin{aligned} v_k^{l(r^\bullet)} := v_k^{l(r^\bullet)} \quad + \quad \left\lceil \frac{1}{n^{l(r^\bullet)}} \max \left\{ z^{r^\bullet}_{kk'} - \sum_{r' \in R^A \setminus R^B \cap \{r^\bullet\}} \left(n^{l(r')} v_k^{l(r')} - \sum_{i \in I_{r'}} x_{ik} \right) \right. \right. \\ - \sum_{r' \in R^B \setminus R^A \cap \{r^\bullet\}} \left(n^{l(r')} v_k^{l(r')} - \sum_{j \in J_{r'}} y_{jk} \right) \\ \left. \left. - \sum_{r' \in R^B \cap R^A \cap \{r^\bullet\}} \left(n^{l(r')} v_k^{l(r')} - \sum_{i \in I_{r'}} x_{ik} - \sum_{j \in J_{r'}} y_{jk} \right), 0 \right\} \right\rceil . \end{aligned}$$

Step 6: If the solution found is feasible, the cost of the solution is given by the objective function (51). Otherwise set the cost to a very large value and stop.

3.1.3 Algorithm for P(e)

We now propose an heuristic to find a feasible solution of problem P(e).

Algorithm for P(e)

Step 1: (Initialisation)

1.1 Set $l(r) := \arg\max_{l \in L_r} \{n^l\}$ for all $r \in R^P$ and $r^\bullet := \arg\max_{r \in R^P} \{\alpha_r\}$.

1.2 Define $P = \{p_1, p_2, ..., p_{|P|}\}$ a subset of K defined as the set of routers' locations in the POP.

1.3 For all $k \in K$ set $h_k := \left\lceil \frac{|P|-1}{n^{l(r^\bullet)}} \right\rceil$.

1.4 Find a solution using algorithms HFS and HSS.

Step 2: (Local search)

Perform a local search on vector h in order to find a local minimum. Each move of the search consists of increasing h_k by one if the router capacity constraints are not respected at router k for the solution found using algorithms HFS and HSS. At each iteration of the search, we determine the best move according to the objective function (5).

We now proceed to the presentation of the main features of the tabu search algorithm.

3.2 The TS Algorithm

Each move of TS consists of modifying the state of a given location in the current solution. At each iteration of the search, we determine the best move while taking into account the tabus as well as the aspiration criterion. The chosen site is declared tabu for a number of iterations that is randomly determined according to a uniform discrete distribution on the interval $[a, b]$.

The aspiration criterion states that if the use of tabu site k allows us to discover a solution better than any other found so far, we may remove the tabu from site k.

We now proceed to a detailed description of TS.

Algorithm TS

Step 1: (Initial solution)

1.1 For all $k \in K$, if for a value of $t \in T$, $\overline{u}_k^t = 1$ set $e_k := t$.

1.2 Define $Q = \{k \in K : \sum_{t \in T} \overline{u}_k^t = 0\} = \{q_1, q_2, ..., q_{|Q|}\}$.

1.3 For $n := 1$ to $|Q|$ do

Set $e_{q_i} := \arg\max_{t \in T}\{m^t\}$ for $i := 1, ..., n$, $e_{q_i} := 0$ for $i := n+1, ..., |Q|$ and find an initial solution by solving P($\mathbf{e}$). If the solution costs less than that of the best solution obtained so far, update this best solution.

Repeat Steps 2 to 3 for Max_Iter iterations

Step 2: (Exploring the neighborhood)

2.1 Determine the best move while taking into account the tabus and the aspiration criterion. For each move $\mathbf{e} \rightarrow \mathbf{e}'$, which modifies the state of a given site in the current solution, we solve P($\mathbf{e}'$) as describe in Section 3.1. The cost of a solution is given by the objective function (5).

2.2 Determine the number of iterations according to a uniform distribution on the interval $[a, b]$ for which the chosen site is tabu.

Step 3: (Best solution update)

If the current solution cost is less than the best solution found so far, update this best solution.

Step 4: (Interface cards costs optimization)

In Steps 2 and 3 the interface cards costs are not optimized. Therefore, we perform here a local search on vector $\mathbf{v}$ to optimize these costs.

4 Numerical Results

In this section, we present a study of the performance of the proposed algorithm TS. The algorithm TS was implemented in the C language on a SunFire 4800 workstation (4 CPU, 900 MHz and 4 GB of RAM). For the tests, two link rates (i.e., OC-3, OC-12) are used in the access network, two in the backbone network (i.e., OC-12, OC-48) and one in the inter-router network (i.e., OC-192). The cost of installing a link is \$100, the costs of the router types are given in Table 1 (including the installation costs) and the costs of the interface card types in Table 2 (including the installation costs). Moreover, for the tests, the single inter-router link failure scenarios are considered as well as the OSPF routing mechanism.

For the tests, the Max_Iter parameter of the algorithm TS is set to 100 and the interval of the number of iteration that a site can be tabu to $[1, 5]$ because the best solutions was found within 100 iterations and with this interval. Moreover the maximum number of routers that can be installed in the POP is set to 5.

Table 1: Characteristics of the router types

Type	A	B
Number of slots	7	14
Switch fabric capacity [Gbps]	80	160
Cost [\$]	15 000	25 000

Table 2: Characteristics of the interface card types

Type	Technology	Number of ports	Cost [\$]
A	OC-3	4	3 200
B	OC-3	8	5 200
C	OC-12	4	10 200
D	OC-12	8	15 200
E	OC-48	4	30 200
F	OC-192	1	50 200

TS results were compared to a lower bound (denoted LB) obtained as follows. The bound LB was obtained by using the CPLEX Mixed Integer Solver [8] to solve P without constraints (19) to (31), without integrality constraints on variables $\mathbf{x}$ and $\mathbf{y}$ and without the flow variables. The default settings of CPLEX are used and the branch-and-bound node limit was set to 200 000 and the upper limit on the size of the tree to 100 MB. We have tried to consider the flow variables and constraints (19) to (31) in the LB formulation but we did not succeed to solve it using CPLEX for real-size problems.

For the evaluation of the performance of TS, 40 design problems are generated randomly. For each client, the number of access links is randomly selected among 1×OC-3, 2×OC-3, 1×OC-12 and 2×OC-12 and for each O-POP, the number of OC-12 links is randomly selected in the interval [0,4] as well as the number of OC-48, with a minimum of one OC-12. Moreover, for each problem, the bandwidth requirement is generated randomly in the interval from 0 to 5 Mbps between each pair of clients, from 0 to 10 Mbps between each pair of O-POPs and from 0 to 5 Mbps between each client and O-POP.

The results are presented in Table 3. In this table, the first column presents the problem name, columns 2 and 3 present respectively the number clients $|M|$ and the number of O-POPs $|N|$, and columns 4 and 5 present respectively the number of access links $|I|$ and the number of backbone links $|J|$. Column 6 presents the lower bound value or the best bound found by the branch-and-bound algorithm if the node limit (NL) or tree size limit (TL) is reached, and column 7 contains the CPU execution time to find the lower bound. Columns 8 to 10 present respectively the solution found by algorithm TS, the CPU execution time to find it and the GAP that indicates the percentage gap between the heuristic solution and the lower bound LB (with respect to the value of the lower bound).

The following observations can be made from Table 3. First, TS finds solutions close to optimality for the problems considered (within 1.16%, on average, from the lower bound) and the mean CPU time is 19.3 seconds. Moreover, the reader should point out that the heuristic finds the optimal solution seven times (i.e., on average, TS find the optimal solution 17.5% of the time for the design problems considered).

5 Conclusions

In this paper, we have presented a mathematical programming model for the POP design problem in IP networks. This problem consists of selecting the number of routers and their types, selecting the interface

Table 3: Computational results

Problem	$\|M\|$	$\|N\|$	$\|I\|$	$\|J\|$	LB		TS		
					Value [$]	CPU [sec]	Value [$]	CPU [sec]	GAP [%]
P1	10	5	13	10	101 300	0.1	103 300	0.7	1.97
P2	20	5	28	22	176 800	0.2	181 800	1.2	2.83
P3	30	5	43	20	188 300	0.1	188 300	2.0	0.00
P4	40	5	59	15	179 600	0.3	179 600	3.5	0.00
P5	50	5	68	31	393 600	6.7	398 600	6.4	1.27
P6	60	5	80	21	348 600	11.6	353 600	9.2	1.43
P7	70	5	108	23	370 400	6.9	377 400	15.4	1.89
P8	80	5	113	26	386 400	12.9	393 400	24.1	1.81
P9	90	5	136	21	413 600	15.2	415 600	32.2	0.48
P10	100	5	140	23	446 600	48.9	451 600	43.0	1.12
P11	10	10	15	51	282 000	0.2	284 000	0.8	0.71
P12	20	10	30	29	211 100	0.1	218 100	1.5	3.32
P13	30	10	38	38	235 000	0.2	235 000	2.5	0.00
P14	40	10	58	42	421 900	15.4	428 900	5.3	1.66
P15	50	10	73	40	473 600	20.2	475 600	8.5	0.42
P16	60	10	82	36	400 900	18.9	405 900	12.7	1.25
P17	70	10	91	31	414 500	29.1	421 500	18.4	1.69
P18	80	10	118	37	483 800	37.8	490 800	30.2	1.45
P19	90	10	131	39	512 500	15.2	517 500	38.3	0.98
P20	100	10	139	45	NL(765 850)	393.1	775 500	54.4	1.26
P21	10	15	17	49	238 800	0.2	238 800	1.1	0.00
P22	20	15	28	53	444 600	4.6	451 600	2.4	1.57
P23	30	15	48	58	494 700	13.4	494 700	4.3	0.00
P24	40	15	63	55	494 300	136.8	501 300	7.7	1.42
P25	50	15	71	54	446 800	20.7	446 800	8.7	0.00
P26	60	15	83	56	567 200	115.5	574 200	16.4	1.23
P27	70	15	95	53	489 700	11.8	489 700	24.8	0.00
P28	80	15	109	53	516 500	4.2	521 500	34.2	0.97
P29	90	15	124	56	NL(776 000)	380.3	779 900	47.1	0.50
P30	100	15	137	54	NL(798 150)	439.6	806 400	61.4	1.03
P31	10	20	14	99	694 400	83.5	701 400	2.0	1.01
P32	20	20	32	81	580 800	11.1	585 800	3.6	0.86
P33	30	20	42	74	579 300	73.6	581 300	5.9	0.35
P34	40	20	56	78	618 500	54.6	618 500	9.4	0.00
P35	50	20	76	71	588 000	7.7	595 000	14.0	1.19
P36	60	20	81	72	595 800	84.9	600 800	20.9	0.84
P37	70	20	101	99	NL(972 850)	487.7	1 003 100	30.0	3.11
P38	80	20	116	86	NL(891 100)	433.7	897 900	38.8	0.76
P39	90	20	132	76	NL(906 750)	550.7	923 900	52.8	1.89
P40	100	20	144	89	NL(994 400)	594.5	1 033 700	76.9	3.95
TS mean results							503 557.5	19.3	1.16

card types, connecting the access and the backbone links to the ports and selecting the link types between the co-located routers. The model also consider the routing of the IP traffic within the POP and all failure scenarios of interest to the network planner. The objective is to minimize the cost of the POP.

All the heuristic results were compared to a proposed lower bound obtained by relaxing a subset of the original constraints in the model. These results have shown that the tabu based heuristic produces solutions close to the lower bound for the 40 test problems considered. As a result, the heuristic proposed is recommended for large-size POP network design.

The following directions for further research are open. First, it would be interesting to consider the POP design problem jointly with the access and backbone network design problem [1, 2, 3]. Second, it would be interesting to explore an exact method in order to solve real-size problems to optimality.

References

[1] Chamberland, S., Sansò, B. and Marcotte, O. "Topological Design of Two-Level Telecommunication Networks with Modular Switches," Operations Research, vol. 48, pp. 745-760, 2000.

[2] Chamberland, S. and Sansò, B. "Topological Expansion of metropolitain Area Networks," Networks, vol. 36, pp. 210-224, 2000.

[3] Chamberland, S. and Sansò, B. "On the Design Problem of Multitechnology Networks," INFORMS Journal on Computing, vol. 13, no. 3, pp. 245-256, 2001.

[4] Chung, S.H., Myung, Y.S. and Tcha, D.W. "Optimal Design of Distributed Network with a Two-Level Hierarchical Structure," European Journal of Operational Research, 62, 1992, 105-115.

[5] Glover, F. and Laguna, M. Tabu Search. Boston, MA: Kluwer Academic Publisher, 1997.

[6] Jonker, R. and Volgenant, T. "A Shortest Augmenting Path Algorithm for Dense and Sparse Linear Assignment Problems," Computing, vol. 38, no. 4, pp. 325-340, 1997

[7] Nemhauser, G.L. and Wolsey, L.A. Integer and Combinatorial Optimization. New-York: John Wiley and Sons Inc., 1988.

[8] ILOG Inc., Using the CPLEX Callable Library and CPLEX Mixed Integer Library. Incline Village: ILOG Inc., 2000.

[9] Moy, J. "OSPF Version 2," IETF RFC 2328, www.ietf.org, 1998.